It follows from (3.1) and (3.3) that the adiabatic equation for an isomagnetic step takes the following form in the pV plane (V is specific volume):

$$p_2 = p_1(V_1\eta - V_2)/(\eta V_2 - V_1) - 2m_H H(V_2 - V_1)/MV_1V_2\ (\eta V_2 - V_1),\ \eta = (\gamma + 1)/(\gamma - 1). \quad (3.4)$$

The mass flux through the surface of discontinuity is given by the following formula, as in gasdynamics:

$$m^2 = (p_2 - p_1)/(V_1 - V_2).$$

It follows from (3.4) that the adiabatic curve in that case passes through the point p_1V_1 and has the same asymptotes as does the Hugoniot adiabatic and lies above the latter for $V_2 < V_1$, but below it for $V_2 > V_1$.

The author is debted to V. V. Gogosov for direction in this work.

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CURRENT AND ENERGY AMPLIFICATION IN A PLANAR CUMULATIVE MAGNETIC GENERATOR WITH FLUX DIFFUSION

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UDC 538.4

§1. Magnetic-field compression in a conducting space (magnetic cumulation) increases the current and the magnetic-field energy; two cases are of interest here: 1) given an initial current I_0 and a load inductance L, select an initial circuit inductance L_0 such as to give the largest final current I; and 2) given the initial nergy U_0 and load L, select L_0 such as to obtain the largest energy U at the end.

Generators of the first type are used to produce very strong magnetic fields and may be called field generators; those of the second type are similarly called energy generators. The two types differ substantially in initial conditions: the initial current is preset in a field generator, and the energy is $U_0 \sim L_0$, while in an energy generator the initial energy is preset, and $I_0 \sim L_0^{-1/2}$. A field generator may be characterized via the current amplification factor

$$i = I/I_0 = (L_0/L)LI/L_0I_0 = \lambda\varphi, \quad (1.1)$$

where $\lambda = L_0/L$ represents the circuit change, and $\varphi = LI/L_0I_0$ is the proportion of the magnetic flux retained in the generator. An energy generator may be characterized by the energy amplification factor

$$\varepsilon = LI^2/L_0I_0^2 = \lambda\varphi^2. \quad (1.2)$$

§2. The quantity φ is a major characteristic of such a generator, as it is dependent on the design, and particularly on the conductivity σ of the material and the field-compression time. Also, the leakage of the flux

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into the conductor is dependent on the initial field distribution at the generator walls, which itself is determined by the time needed to pump the generator from the current source. A scheme has been suggested [1, 2] for calculating the loss of flux by diffusion in a CM generator, and equations were derived for the flux leakage into the conductor for magnetic accumulation in narrow cavities. In the case of a generator with current leads of constant width, it was shown that the flux variation corresponds to the following universal self-modeling state at the end of the compression:

$$\varphi = (1 + 2\sqrt{\pi/\mu}\sqrt{1-t} + (4/\mu)(1-t))\varphi_*, \tag{2.1}$$

which is governed by the single constant φ_*, which itself is dependent on the magnetic Reynolds number $\mu = 4\pi\sigma a^2 D/c^2 l_0$ and the initial field distribution in the conductors. Here the time has been referred to the flux compression time l_0/D, while a is the width of the generator cavity, l_0 is the initial length, and D is the speed of the piston that compresses the flux. The conductivity σ and speed D are assumed constant. Numerical calculations show that (2.1) describes the flux leakage satisfactorily for $1 - t \leq 1/16$, while φ_* is only slightly dependent on the shape of the pumping-current pulse and is determined in the main by the thickness s of the skin layer set up in the cavity walls, which allows us to choose the initial field distribution $B_0(x)$ in a form convenient for computation and for obtaining explicit formulas for φ_*. For instance, for $B_0 = e^{-x/s}$ we have

$$\varphi_*(\mu, s) = \frac{2\mu s^3}{(2s-1)^3} B_*\left(\frac{1}{\mu s^2}\right) + \left(\left(1 - \frac{\mu}{8}\right)\frac{8s^2}{(2s-1)^2} - \frac{\mu s^2}{(2s-1)^3} - \frac{1+8/\mu}{2s-1}\right) B_*\left(\frac{4}{\mu}\right) + 4\sqrt{\frac{\mu}{\pi}}\left(\frac{1}{\mu}\frac{1}{2s-1} - \frac{s^2}{(2s-1)^2}\right), \tag{2.2}$$

where s has been referred to the width a of the slot; $B_*(z) = e^z(1 - \Phi(\sqrt{z}))$; and $\Phi(\sqrt{z})$ is the probability integral. The function $B_*(z^2)$ has been tabulated [3]. For rapid pumping we have $s \to 0$, and $\varphi_* \to \varphi_{*0} = (1 + 8/\mu)B_*(4/\mu) - 4/\sqrt{\pi\mu}$; while for slow pumping we have $s \to \infty$, and then $\varphi_* \to \varphi_*^0 = \mu/4 - \sqrt{\mu/\pi} + 2(1 - \mu/8)B_*(4/\mu)$, and (2.2) can be rewritten as

$$\varphi_*(\mu, s) = \frac{\varphi_{*0}}{1-2s} + \frac{4s^2}{(2s-1)^2}\varphi_*^0 + \frac{\mu s^2}{(2s-1)^3}\left(B_*\left(\frac{1}{\mu s^2}\right) - B_*\left(\frac{4}{\mu}\right)\right) - \frac{\mu s^2}{(2s-1)^2}\left(1 - B_*\left(\frac{1}{\mu s^2}\right)\right), \tag{2.3}$$

from which we have the asymptotic formula

$$\varphi_*(\mu, s) = (1 + 2s)\varphi_{*0}, \quad s \ll \mu^{-1/2},$$

$$\varphi_*(\mu, s) = \varphi_*^0 - \frac{1 - \varphi_*^0}{s} + \frac{\varphi_*^0 - \varphi_{*0}}{2s}, \quad s \gg \mu^{-1/2}.$$

Formulas (2.2) and (2.3) give an indeterminacy for s = 1/2, which is resolved to give

$$\varphi_*(\mu, 1/2) = (1 - 64/3\mu^2)B_*(4/\mu) - (4/3)(1/\sqrt{\pi\mu})(1 - 8/\mu).$$

§3. By considering the flux at time t, we can derive the flux at the load for a generator of simple form whose length is $l = (1 - t)l_0$, i.e., we can determine the mode of operation for the following compression factor:

$$\lambda = 1/(1 - t). \tag{3.1}$$

To avoid the need for computation, we assume that $\lambda \geq 16$, and then substitute (3.1) into (2.1) to get the flux at the load; then from (1.1) and (1.2) we calculate the characteristics of the generators:

$$i = (1 + 2\sqrt{\pi/m} + 4/m)\lambda\varphi_*; \tag{3.2}$$

$$\varepsilon = (1 + 2\sqrt{\pi/m} + 4/m)^2\lambda\varphi_*^2. \tag{3.3}$$

Here $m = \lambda\mu$ is the magnetic Reynolds number for the load, which is the ratio of the time for the flux to diffuse from the load to the time for the last current doubling in an ideal generator. Practical interest attaches mainly to generators with μ large and $\lambda \geq 16$, so $m \gg 1$, and (3.2) and (3.3) amount to

$$i = m\varphi_*(m/\lambda, s)/(m/\lambda); \tag{3.4}$$

$$\varepsilon = m\varphi_*^2(m/\lambda, s)/(m/\lambda). \tag{3.5}$$

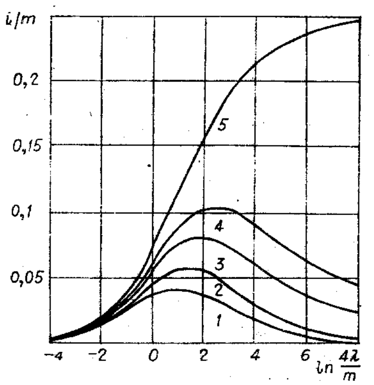

Fig. 1

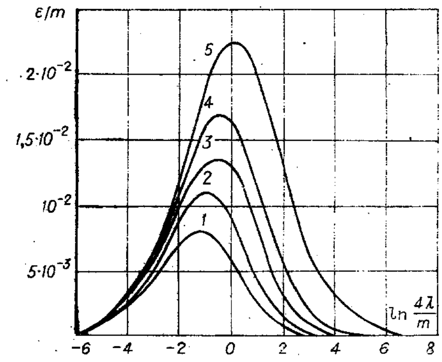

Fig. 2

At present, CM generators are driven in the main from capacitor banks, with the pumping time t_0 a quarter of the discharge period.

The thickness of the skin layer increases as $\sqrt{t_0}$ during the pumping; the capacitor bank is specified for an energy generator, while the circuit inductance is proportional to λ, and then $t_0 \sim \sqrt{\lambda}$, and

$$s = \delta\sqrt[4]{\lambda}, \qquad (3.6)$$

where δ is the thickness of the skin layer arising on direct discharge of the bank through the load. The same relation applies also for a field generator if the initial current is maintained by altering the potential difference across the capacitor bank without altering the capacitance.

We substitute (2.2) into (3.4) and (3.5) and use (3.6) to calculate the current and energy amplification factors on compressing a magnetic field in a simple generator; Figs. 1 and 2 show the results for $\delta m^{1/4} = 0$; 0.05; 0.5; 2; ∞ as curves 1-5, respectively.

The current amplification factor is clearly very much dependent on the initial field distribution, since it initially increases with λ, but falls very slowly after the peak. If the pumping is slow, i reaches the asymptote $i = m/4$. The peak i shifts to larger λ/m as $\delta m^{1/4}$ increases, while the value changes from $4.2 \cdot 10^{-2}$m to 0.25m. The least value for λ corresponding to peak i for rapid pumping is 0.7m.

If the capacitor voltage remains unchanged in a field generator, the capacitance must be increased in proportion to λ in order to maintain the initial current; then $t_0 \sim \lambda$ and $s = \delta\sqrt{\lambda}$, i.e., the thickness of the skin layer is much larger than that for a fixed capacitor bank, and the current for a given δ attains larger values and falls even more slowly after the peak. However, $i \leq m/4$ in all cases.

Further, ε has a clear peak for all $\delta m^{1/4}$, and this lies in the range $0.3136 \leq 4\lambda/m \leq 1.1664$, while the height of the peak increases with $\delta m^{1/4}$ from $8 \cdot 10^{-3}$ m to $2.22 \cdot 10^{-2}$ m; this is readily understood in physical terms. If λ is small, the flux loss is also small, but ε is also small because λ is small. If λ is large, the working time is long, and the flux loss increases, and hence ε falls.

It is fairly simple to obtain $m \sim 10^3$, $\delta m^{1/4} \sim 0.1$ in experiments; larger values are difficult to obtain, since this requires excessively small loads and very large, low-voltage capacitor banks. Under such conditions, one expects at most a 12-fold increase in the energy for $\lambda = 140$ and a 70-fold increase in the current for $\lambda = 1300$. This shows that magnetic cumulation in a planar generator cannot be expected to provide an energy increase by more than an order of magnitude. We get from (3.3) that such a generator has little to recommend it on energy grounds for $m < 30$ with slow pumping or with $m < 80$ for fast pumping. Detailed numerical calculations show that the maximum ε are 1.3 and 1.92 for m of 8 and 24, respectively, in the case of slow pumping, these values being obtained for λ of 1.6 and 8, respectively.

§4. The equation of [2] for the flux loss by diffusion in magnetic cumulation between conductors of variable width can be solved analytically when the width is $z(y) = e^{\alpha y}$ $(-1 \leq y \leq 0)$ and the length of the load is $l = (1/\alpha)l_0$; in that case, the referred inductance of the generator is $L(t) = e^{-\alpha t}$, while $\lambda = e^{\alpha}$, and methods similar to those described in [1] then give us the equation for the magnetic field at the load:

$$\frac{d^2B}{dt^2} - \left(\frac{4}{\mu} + 2\alpha\right)\frac{dB}{dt} + \alpha^2 B = -\frac{2\alpha}{\sqrt{\pi\mu}}\frac{1}{\sqrt{t}} - \frac{2\alpha}{\mu} f_0(t) +$$

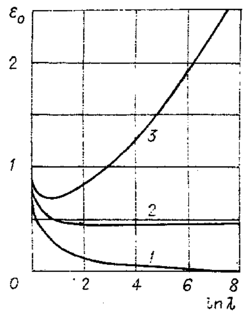

Fig. 3

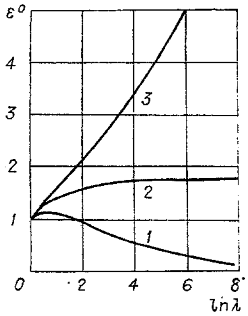

Fig. 4

$$+\frac{2}{\mu}f_0'(t)-\frac{8}{\mu\sqrt{\pi\mu}}\frac{d}{dt}\int_0^t\frac{f_0(\xi)\,d\xi}{\sqrt{t-\xi}}, \tag{4.1}$$

$$f_0(t)=\frac{2}{\sqrt{\pi}}\int_0^\infty B_0'\left(\xi\frac{2\sqrt{t}}{\sqrt{\mu}}\right)e^{-\xi^2}d\xi,$$

which may be solved subject to the initial conditions $B(0) = 1$, $B'(0) = \alpha + (2/\mu) f_0(0)$; the current gain is then $i = B(1)$, while the energy amplification factor is $\varepsilon = (1/\lambda)i^2 = e^{-\alpha}B^2(1)$. We solve (4.1) to get for fast pumping that

$$i_0=\frac{\sqrt{1+m}-1}{\sqrt{1+m}}\exp\left(\frac{(\sqrt{1+m}-1)^2}{m}\alpha\right)+\frac{\sqrt{1+m}+1}{2\sqrt{1+m}}B_*\left(\frac{(\sqrt{1+m}+1)^2}{m}\alpha\right)+\frac{\sqrt{1+m}-1}{2\sqrt{1+m}}B_*\left(\frac{(\sqrt{1+m}-1)^2}{m}\alpha\right) \tag{4.2}$$

and for slow pumping that

$$i^0=\frac{\sqrt{1+m}+1}{\sqrt{1+m}}\exp\left(\frac{(\sqrt{1+m}-1)^2}{m}\alpha\right)+\frac{\sqrt{1+m}-1}{2\sqrt{1+m}}B_*\left(\frac{(\sqrt{1+m}+1)^2}{m}\alpha\right)-\frac{\sqrt{1+m}+1}{2\sqrt{1+m}}B_*\left(\frac{(\sqrt{1+m}-1)^2}{m}\alpha\right). \tag{4.3}$$

Here $m = \alpha\mu$ is the magnetic Reynolds number for the load.

It is clear that this altered shape substantially affects the operation. If λ is large, the terms containing B_* vanish in (4.2) and (4.3), and the current increases with λ for any m:

$$i_0=\frac{\sqrt{1+m}-1}{\sqrt{1+m}}\lambda^{\frac{(\sqrt{1+m}-1)^2}{m}},$$

$$i^0=\frac{\sqrt{1+m}+1}{\sqrt{1+m}}\lambda^{\frac{(\sqrt{1+m}-1)^2}{m}},\quad \lambda\gg1.$$

Further, the energy-increase factors are then

$$\varepsilon_0=\left(\frac{\sqrt{1+m}-1}{\sqrt{1+m}}\right)^2\lambda^{\frac{2(\sqrt{1+m}-1)^2}{m}-1},$$

$$\varepsilon^0=\left(\frac{\sqrt{1+m}+1}{\sqrt{1+m}}\right)^2\lambda^{\frac{2(\sqrt{1+m}-1)^2}{m}-1},\quad \lambda\gg1$$

and are substantially dependent on the load. If $m < 8$, ε falls as λ increases, while if $m > 8$ the two increase together. If $m = 8$, ε tends asymptotically to a constant having the value 4/9 for rapid pumping or 16/9 for slow pumping.

Figure 3 shows results calculated for ε for rapid pumping in such a generator for m of 3, 8, and 15 (curves 1-3). Figure 4 shows similar calculations for slow pumping for the same m. Clearly, i and ε increase considerably with m, and $i \to \lambda$, $\varepsilon \to \lambda$ for $m \gg 1$, as should be the case for a generator with no flux loss.

Therefore, conductor shaping can substantially reduce the loss of flux by diffusion and considerably improve the performance of a CM generator. There are two reasons for this: first, the shaped conductors allow one to provide a given λ with a shorter generator, which reduces the working time and thus reduces the flux loss; and secondly, the field in such a generator is inhomogeneous: it is large near the point where the conductors meet and weak in the rest of the generator. This field distribution means that the flux losses in the wide part and in the load can be neglected for almost all the flux compression time, with only a minor correction for the small zone near the junction and also for the short period required to compress the field in this zone. Of course, the specifications for the contact in that case are very much more severe, since even minor irregularities on the conductors result in trapping the strong field and thus large contact losses.

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DAMAGE PRODUCED IN GLASS BY METEORITE IMPACT

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UDC 523.51

The effects of meteorites on transparent brittle materials are important in long-term operation of optical systems in space, such as windows, solar-battery coatings, and so on. Since the material is brittle, the damage on impact differs from that for a plastic metal.

Exposure of specimens in space shows [1] that the surface damage is particularly important, since the probability of encountering a large particle is low. Sometimes, however, interest attaches to the possibility that the specimen will be entirely destroyed. Studies have been made [2] of the effects of particles of micron size on glass and quartz for particle masses m of approximately 10^{-10}–10^{-12} g traveling at speeds v of 2-14 km/sec. Data are available only from isolated tests [3] for larger particles, so it is desirable to compare [2] with a fairly wide range of evidence for large particles in order to elucidate the scope for scale simulation and also to refine our picture of the process.

The present experiments were performed under laboratory conditions by means of explosions [4]; we used spherical steel particles accelerated to v = 5-12 km/sec and having diameters d = 0.7-2.3 mm ($m \sim 10^{-3}$–$5 \cdot 10^{-2}$ g). The specimens were glass disks (optical crown glass) with polished surfaces; a specimen was attached to a metal holder by a flat clamp at the edge acting via a damping ring; the side surface remained free, while the diameter was 115-255 mm, having a thickness δ of 8-20 mm. For comparison, several tests were performed with quartz specimens. The system prevented the explosion products from affecting the specimen; it was not necessary to ensure that the particles struck the center of the disk. A few experiments were done with particles in the range d = 0.1-0.3 mm ($m \sim 3 \cdot 10^{-6}$– 10^{-4} g), which were accelerated in a vacuum chamber to 5-13.5 km/sec, the final size being determined within ± 10%. In addition, measurements were made with glass particles.

Figure 1 shows a photograph of a specimen of diameter 115 mm and δ = 15 mm after the experiment (d = 0.75 mm and v = 10 km/sec for the particle). A radial ringed structure in the cracks is clear, and this is the same for any speed of impact. The diameter of this zone is $D \gg d$, and is close to the sizes observed on impact on rocks [5, 6], but in the case of glass the material is ejected only from a central part of size D_1

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